文章编号　1672-6634(2020)06-0055-05　　DOI　10.19728/j.issn1672-6634.2020.06.007

受振幅阻尼和热噪声共同影响的克尔介质主方程的解析解

孟祥国　刘钧毅

(聊城大学 山东省光通信科学与技术重点实验室、物理科学与信息工程学院,山东 聊城 252059)

摘　要　根据连续变量热场纠缠态表象导出的量子系统(实模)和环境(虚模)间的算符对应关系,解析求解了受振幅阻尼和热噪声同时影响的克尔介质量子主方程,并给出了其解析解的无限维广义克劳斯算符和表示.此外,还证明了广义克劳斯算符满足归一化条件.

关键词　热场纠缠态表象;噪声克尔介质;广义克劳斯算符

中图分类号　O413.1　　**文献标识码**　A

由于强克尔介质非线性系统在非破坏性测量[1]、量子计算[2]和单粒子探测[3]等方面有着重要应用,但环境噪声对克尔介质非线性强度的减弱作用又是不可避免的[4-6],故噪声克尔介质中光场的非线性相互作用在目前受到广泛关注.如,文献[7]和[8]分别讨论振幅阻尼和热环境影响下克尔介质中系统的密度算符、维格纳函数以及光子数分布随时间的解析退相干演化规律;文献[9]研究了在非线性克尔介质和参量振荡器作用下相干态的时间演化,并分析了不同哈密顿量参数下的概率振幅、自相关函数和 Husimi 分布函数;而文献[10]考察了耗散克尔介质中相干态的传输特性,发现非线性相位噪声限制了具有“类克尔”非线性耗散单模玻色通道上以相位变量传输经典信息的能力.

然而,以往的研究主要集中讨论单一噪声(如振幅阻尼[7]、热噪声[8]等)对克尔介质中光场的非线性作用的影响.作为重要的推广,Stobińska 及其合作者首先把克尔介质“浸”在振幅阻尼和热噪声同时存在的环境,并通过数值求解和分析维格纳函数满足的福克-普朗克方程,揭示了共同存在的两种噪声对克尔介质中光场的影响[11].与 Stobińska 的数值求解方法不同,本文利用连续变量热场纠缠态表象[12,13],解析求解了受振幅阻尼和热噪声同时影响的克尔介质主方程,并给出了含时密度算符的无限维克劳斯算符和表示.

1　受两种噪声影响的克尔介质的量子主方程

根据马尔科夫近似理论,当克尔介质受振幅阻尼和热噪声同时影响时,系统的含时密度算符在相互作用表象中满足量子主方程[11]

$$\begin{aligned}\frac{d\rho_t}{dt} =& -\mathrm{i}\kappa[(a^{\dagger}a)^2,\rho_t]+\Gamma(2a\rho_t a^{\dagger}-a^{\dagger}a\rho_t-\rho_t a^{\dagger}a)\\&+\Gamma N(a\rho_t a^{\dagger}+a^{\dagger}\rho_t a-a^{\dagger}a\rho_t-\rho_t aa^{\dagger}),\end{aligned}\tag{1}$$

式中 ρ_t 为 t 时刻系统的密度算符,$a(a^{\dagger})$ 为系统的湮灭(产生)算符,Γ 为振幅阻尼系数,$N=1/(e^{\hbar}-1)$ 为热库的平均光子数,参数 $\hbar$,ω,k 和 T 分别为普朗克常数、谐振子频率、玻尔兹曼常数和热场的温度.κ 为与克尔介质的非线性极化率 $\chi^{(3)}$ 有关的非线性常数.特殊地,当 $N\to 0$ 且 Γ 为有限值时,主方程(1)变成了受振幅阻尼噪声影响的克尔介质的量子主方程[7]

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}t}=-\mathrm{i}\kappa[(a^{\dagger}a)^2,\rho_t]+\Gamma(2a\rho_t a^{\dagger}-a^{\dagger}a\rho_t-\rho_t a^{\dagger}a).\tag{2}$$

收稿日期:2020-03-20

基金项目:国家自然科学基金项目(11347026);山东省自然科学基金项目(ZR2016AM03,ZR2017MA011)资助

通讯作者:孟祥国,男,汉族,博士,教授,研究方向:量子光学与量子信息,E-mail:mengxiangguo1978@sina.com.

当 $\Gamma \to 0$ 且 $N \to \infty$时,这样 $\Gamma N\ (\equiv \varepsilon\ )$为有限值,主方程(1)变成受热噪声影响的克尔介质的量子主方程[8]

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}t} = -\mathrm{i}\kappa[(a^\dagger a)^2,\rho_t] + \varepsilon(a\rho_t a^\dagger + a^\dagger \rho_t a - a^\dagger a\rho_t - \rho_t a a^\dagger)\ , \tag{3}$$

而当 $\Gamma \to 0$ 且 $N \to 0$ 时,主方程(1)退化为描述克尔介质的主方程,即 $\mathrm{d}\rho_t/\mathrm{d}t = -\mathrm{i}\kappa[(a^\dagger a)^2,\rho_t]$.因此,利用热场纠缠态表象去分析受两种噪声影响的克尔介质中密度算符的解析演化是非常有意义的.

2 热场纠缠态表象

根据 Umezawa-Takahash 热场动力学理论,在扩展的福克空间中,文献[12,13]引入了描述系统与环境之间量子纠缠的热场纠缠态 $|\tau\rangle$,其具体表达式为

$$|\tau\rangle = \exp(-\frac{|\tau|^2}{2} + \tau a^\dagger - \tau^* b^\dagger + a^\dagger b^\dagger)\,|0\tilde{0}\rangle = D(\tau)\,|\tau = 0\rangle, \tag{4}$$

式中 $D(\tau)$ 为平移算符,$b^\dagger$ 为环境(虚)模的产生算符,与系统(实)模的产生算符 $a^\dagger$ 相对应,且存在对易关系 $[b,b^\dagger] = 1$ 和 $[b,a^\dagger] = 0$.当把系统和环境的湮灭算符 a,b 分别作用到态 $|\tau\rangle$,可导出如下本征方程

$$\begin{gathered}(a - b^\dagger)\,|\tau\rangle = \tau\,|\tau\rangle,\ (a^\dagger - b)\,|\tau\rangle = \tau^*\ |\tau\rangle,\\ \langle\tau|\,(a^\dagger - b) = \tau^*\langle\tau|,\ \langle\tau|\,(a - b^\dagger) = \tau\langle\tau|.\end{gathered} \tag{5}$$

式(5)表明,纠缠态 $|\tau\rangle$ 恰好为线性组合算符 $a - b^\dagger$ 和 $a^\dagger - b$ 的共同本征态.进一步,利用双模真空投影算符的正规乘积表示,即 $|0\tilde{0}\rangle\langle 0\tilde{0}| = :\exp(-a^\dagger a - b^\dagger b):$(符号 $:\ :$代表正规乘积[14]),以及有序算符内积分法[15],易证纠缠态 $|\tau\rangle$ 具有完备正交性,即

$$\frac{1}{\pi}\int \mathrm{d}^2\tau\,|\tau\rangle\langle\tau| = 1\ ,\quad \langle\tau'|\tau\rangle = \pi\delta(\tau - \tau')\delta(\tau^* - \tau'^*). \tag{6}$$

另外,注意到态 $|\tau = 0\rangle = \exp(a^\dagger b^\dagger)\,|0\tilde{0}\rangle = \sum_{n=0}^{\infty}|n,\tilde{n}\rangle$,这样,我们有

$$a\,|\tau = 0\rangle = b^\dagger\,|\tau = 0\rangle, a^\dagger\,|\tau = 0\rangle = b\,|\tau = 0\rangle, a^\dagger a\,|\tau = 0\rangle = b^\dagger b\,|\tau = 0\rangle, \tag{7}$$

故在纠缠态 $|\tau = 0\rangle$ 下,系统(实模)和环境(虚模)的算符存在如下对应关系

$$a \Leftrightarrow b^\dagger, a^\dagger \Leftrightarrow b, a^\dagger a \Leftrightarrow b^\dagger b\ , \tag{8}$$

利用此对应关系能把方程(1)中的密度算符主方程转化为关于态矢量 $|\tau = 0\rangle$ 的方程.

类似地,引入另一个与态 $|\tau\rangle$ 共轭的热纠缠态 $|\zeta\rangle$,其具体的表达式为

$$|\zeta\rangle = \exp(-\frac{|\zeta|^2}{2} + \zeta a^\dagger + \zeta^* b^\dagger - a^\dagger b^\dagger)\,|0\tilde{0}\rangle = D(\zeta)\,|\zeta = 0\rangle = (-1)^{a^\dagger a}\,|\tau = -\zeta\rangle, \tag{9}$$

它们具有如下完备正交性

$$\frac{1}{\pi}\int \mathrm{d}^2\zeta\,|\zeta\rangle\langle\zeta| = 1\ , \langle\zeta'|\zeta\rangle = \pi\delta(\zeta - \zeta')\delta(\zeta^* - \zeta'^*)\ . \tag{10}$$

3 噪声克尔介质主方程的解析解

把主方程(1)的两端同时作用到态 $|\tau = 0\rangle$ 上,利用算符对应关系(8)并定义态 $|\rho_t\rangle \equiv \rho_t\,|\tau = 0\rangle$,可得

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|\rho_t\rangle &= \{-\mathrm{i}\kappa[(a^\dagger a)^2,\rho_t] + \Gamma(2a\rho_t a^\dagger - a^\dagger a\rho_t - \rho_t a^\dagger a) + \Gamma N(a\rho_t a^\dagger + a^\dagger\rho_t a - a^\dagger a\rho_t - \rho_t a a^\dagger)\}\,|\tau = 0\rangle\\ &= \{-\mathrm{i}\kappa[(a^\dagger a)^2 - (b^\dagger b)^2] + \Gamma(2ab - a^\dagger a - b^\dagger b) + \Gamma N(ab + a^\dagger b^\dagger - a^\dagger a - bb^\dagger)\}\,|\rho_t\rangle,\end{aligned} \tag{11}$$

这样,我们可直接给出态矢量 $|\rho_t\rangle$ 的标准解

$$\begin{aligned}|\rho_t\rangle = \exp\{&-\mathrm{i}\kappa t[(a^\dagger a)^2 - (b^\dagger b)^2] + \Gamma t(2ab - a^\dagger a - b^\dagger b)\\ &+ N\Gamma t(ab + a^\dagger b^\dagger - a^\dagger a - bb^\dagger)\}\,|\rho_0\rangle,\end{aligned} \tag{12}$$

其中 $|\rho_0\rangle \equiv \rho_0\,|\tau = 0\rangle$,$\rho_0$ 为初始的密度算符.引入算符

$$K_+ = a^\dagger b^\dagger,\ K_- = ab,\ K_0 = a^\dagger a - b^\dagger b,\ 2K_z = a^\dagger a + b^\dagger b + 1\ , \tag{13}$$

式中 K_+,K_- 和 K_z 构成 SU(1,1)李代数,具有对易关系 $[K_-,K_+] = 2K_z$,$[K_z,K_\pm] = \pm K_\pm$,而 K_0 为 Casimir 算符,与 SU(1,1)李代数算符 $K_z,K_\pm$ 均对易,即 $[K_0,K_\pm] = [K_0,K_z] = 0$.这样,利用 SU(1,1)李代数算符及其满足的对易关系,可把式(12)改写为

$$\begin{aligned}|\rho_t\rangle &= \exp\{-\mathrm{i}\kappa t K_0(2K_z-1)+\Gamma t(2K_-+1-2K_z)+N\Gamma t(K_++K_--2K_z)\}|\rho_0\rangle \\ &= \exp[(\Gamma+\mathrm{i}\kappa K_0)t]\exp(\lambda_+K_++\lambda_zK_z+\lambda_-K_-)|\rho_0\rangle,\end{aligned} \tag{14}$$

式中

$$\lambda_+=N\Gamma t,\ \lambda_-=(N+2)\Gamma t,\lambda_z=-2[(N+1)\Gamma+\mathrm{i}\kappa K_0]t. \tag{15}$$

进一步，通过利用涉及 SU(1,1)李代数的解纠缠定理，可把纠缠项 $\exp(\lambda_+K_++\lambda_zK_z+\lambda_-K_-)$ 分解为

$$\exp(\lambda_+K_++\lambda_zK_z+\lambda_-K_-)=\exp(\Lambda_+K_+)\exp(2K_z\ln\sqrt{\Lambda_z})\exp(\Lambda_-K_-), \tag{16}$$

式中

$$\Lambda_\pm=\frac{2\lambda_\pm\sinh\varphi}{2\varphi\cosh\varphi-\lambda_z\sinh\varphi},\ \sqrt{\Lambda_z}=\frac{2\varphi}{2\varphi\cosh\varphi-\lambda_z\sinh\varphi},\ \varphi^2=\frac{\lambda_z^2}{4}-\lambda_+\lambda_-. \tag{17}$$

因此，式(14)可表示为

$$|\rho_t\rangle=\exp[(\Gamma+\mathrm{i}\kappa K_0)t]\exp(\Lambda_+K_+)\exp(2K_z\ln\sqrt{\Lambda_z})\exp(\Lambda_-K_-)|\rho_0\rangle. \tag{18}$$

为了把态 $|\tau=0\rangle$ 从式(18)的两端剥离出来，把福克态的完备性关系 $\sum_{m,n=0}^{\infty}|m,\tilde{n}\rangle\langle m,\tilde{n}|=1$ 插入式(18)，并利用等式 $a^{\dagger i}|m\rangle=\sqrt{(m+i)!/m!}|m+i\rangle$，则式(18)变为

$$\begin{aligned}|\rho_t\rangle &= \sum_{l,m,n=0}^{\infty}\frac{\Delta_-^l}{l!}\sqrt{\Delta_z^{(m+n+1)}}\exp[(\Gamma+\mathrm{i}\kappa K_0)t]\times\sum_{k=0}^{\infty}\frac{\Delta_+^k}{k!}(a^\dagger b^\dagger)^k|m,\tilde{n}\rangle\langle m,\tilde{n}|a^l\rho_0a^{\dagger l}|\tau=0\rangle \\ &= \sum_{k,l,m,n}^{\infty}\sqrt{\frac{(m+k)!(n+k)!}{m!n!}\Delta_z^{m+n+1}}\times\frac{\Delta_+^k\Delta_-^l\Pi}{k!l!}|m+k,\widetilde{n+k}\rangle\langle m,\tilde{n}|a^l\rho_0a^{\dagger l}|\tau=0\rangle,\end{aligned} \tag{19}$$

式中参数 $\Delta_\pm$，Δ_z 和 Π 分别为

$$\begin{aligned}&\Delta_+=2N\Gamma t\Theta_{m,n}\sinh\varphi_{m,n},\Delta_-=2(N+2)\Gamma t\Theta_{m,n}\sinh\varphi_{m,n}, \\ &\sqrt{\Delta_z}=2\Theta_{m,n}\varphi_{m,n},\Pi=\exp[(\Gamma+\mathrm{i}\kappa(m-n))t], \\ &\Theta=\frac{1}{2\varphi_{m,n}\cosh\varphi_{m,n}+2t[\Gamma(N+1)+\mathrm{i}\kappa(m-n)]\sinh\varphi_{m,n}}, \\ &\varphi_{m,n}=t\{[\Gamma(N+1)+\mathrm{i}\kappa(m-n)]^2-N(N+2)\Gamma^2\}^{1/2}.\end{aligned} \tag{20}$$

进一步，利用恒等式

$$\langle\tilde{n}|\tau=0\rangle=\sum_{n=0}^{\infty}\langle\tilde{n}|n,\tilde{n}\rangle=|n\rangle, \tag{21}$$

可有

$$\langle m,\tilde{n}|a^l\rho_0a^{\dagger l}|\tau=0\rangle=\langle m|a^l\rho_0a^{\dagger l}\langle\tilde{n}|\tau=0\rangle=\langle m|a^l\rho_0a^{\dagger l}|n\rangle. \tag{22}$$

利用式(22)把式(19)改写为

$$\begin{aligned}|\rho_t\rangle &= \sum_{k,l,m,n}^{\infty}\sqrt{\frac{(m+k)!(n+k)!}{m!n!}\Delta_z^{m+n+1}}\times\frac{\Delta_+^k\Delta_-^l\Pi}{k!l!}|m+k,\widetilde{n+k}\rangle\langle m|a^l\rho_0a^{\dagger l}|n\rangle \\ &= \sum_{k,l,m,n}^{\infty}\sqrt{\frac{(m+k)!(n+k)!}{m!n!}\Delta_z^{m+n+1}}\frac{\Delta_+^k\Delta_-^l\Pi}{k!l!}\times\langle m|a^l\rho_0a^{\dagger l}|n\rangle|m+k\rangle\langle n+k|\tau=0\rangle.\end{aligned} \tag{23}$$

把态 $|\tau=0\rangle$ 从式(23)的左右两端同时去掉，则得到量子主方程(1)的标准解，即

$$\rho_t=\sum_{k,l,m,n}^{\infty}\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!}a^{\dagger k}|m\rangle\langle m|a^l\rho_0a^{\dagger l}|n\rangle\langle n|a^k. \tag{24}$$

上式表明，一旦给定初始态 ρ_0，易给出任意时刻的密度算符 ρ_t，并为进一步分析初始态 ρ_0 的时间演化特性及其非经典性质等提供方便. 实际上，此解也可表示为无限维克劳斯算符和表示，即

$$\rho_t=\sum_{k,l,m,n=0}^{\infty}M_{k,l,m,n}\rho_0M_{k,l,m,n}^\dagger, \tag{25}$$

这是因为(25)式中 $M_{k,l,m,n}=\left(\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!}\right)^{1/2}a^{\dagger k}|m\rangle\langle m|a^l$，和

$$M_{k,l,m,n}^\dagger=\left[\left(\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!}\right)^{1/2}a^{\dagger k}|n\rangle\langle n|a^l\right]^\dagger. \tag{26}$$

虽不是厄米共轭关系，但满足克劳斯算符的归一化条件(见式(33))，故称之为广义克劳斯算符. 特别

地,当 $N\to 0$ 且 Γ 为有限值时,由于 $\lambda_+=0,\lambda_-=2\Gamma t,\lambda_z=-2(\Gamma+\mathrm{i}\kappa K_0)t$,那么我们有

$$\begin{aligned}&\Delta_+=0,\Delta_-=4\Gamma t\Theta_{m,n}\sinh\varphi_{m,n},\\&\sqrt{\Delta_z}=2\Theta_{m,n}\varphi_{m,n},\varphi_{m,n}=[\Gamma+\mathrm{i}\kappa(m-n)]t,\\&\Theta_{m,n}=\frac{1}{2\varphi_{m,n}\cosh\varphi_{m,n}+2t[\Gamma+\mathrm{i}\kappa(m-n)]\sinh\varphi_{m,n}},\end{aligned}\tag{27}$$

式(25)变成仅受振幅阻尼噪声影响的克尔介质的量子主方程的解析解.当 $\Gamma\to 0$ 且 $N\to\infty$时,由于 $\varepsilon=\Gamma N$ 为有限值,这样 $\lambda_+=\lambda_-=\varepsilon t$, $\lambda_z=-2(\varepsilon+\mathrm{i}\kappa K_0)t$,则有

$$\begin{aligned}&\Delta_\pm=2\varepsilon t\Theta_{m,n}\sinh\varphi_{m,n},\sqrt{\Delta_z}=2\Theta_{m,n}\varphi_{m,n},\\&\varphi_{m,n}=t\left[\mathrm{i}2\varepsilon\kappa(m-n)-\kappa^2(m-n)^2\right]^{1/2},\Pi=\exp[\mathrm{i}\kappa(m-n)t],\\&\Theta_{m,n}=\frac{1}{2\varphi_{m,n}\cosh\varphi_{m,n}+2t[\varepsilon+\mathrm{i}\kappa(m-n)]\sinh\varphi_{m,n}}.\end{aligned}\tag{28}$$

因此,式(25)退化为仅受热噪声影响的克尔介质的量子主方程(3)的解析解.而当 $\Gamma\to 0$ 且 $N\to 0$ 时,由于 $\lambda_+=\lambda_-=0$, $\lambda_z=-\mathrm{i}2\kappa K_0 t$,且 $\Delta_\pm=0$, $\sqrt{\Delta_z}=\exp[-\mathrm{i}\kappa t(m-n)]=\Pi^{-1}$,故式(25)为克尔介质主方程的解析解,即

$$\rho_t=\sum_{m,n}^{\infty}\exp[-\mathrm{i}\kappa t(m^2-n^2)]\,|m\rangle\langle m|\rho_0|n\rangle\langle n|.\tag{29}$$

可见,初始态 ρ_0 在克尔介质中不发生退相干效应.

4 克劳斯算符的归一化

利用等式 $|m\rangle=(a^{\dagger m}|0\rangle)/\sqrt{m!}$ 和真空态投影算符的正规乘积表示[16,17],即 $|0\rangle\langle 0|=:\mathrm{e}^{-a^\dagger a}:$,可有

$$\begin{aligned}\sum_{k,l,m,n=0}^{\infty}\mathrm{M}^\dagger_{k,l,m,n}M_{k,l,m,n}&=\sum_{k,l,m,n=0}^{\infty}\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!}a^{\dagger l}|n\rangle\langle n|a^k a^{\dagger k}|m\rangle\langle m|a^l\\&=\sum_{k,l,m,n=0}^{\infty}\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!m!n!}:a^{\dagger l+n}e^{-a^\dagger a}a^{k+n}::a^{\dagger k+m}e^{-a^\dagger a}a^{l+m}:.\end{aligned}\tag{30}$$

把相干态的完备性关系插入式(30)中,得到

$$\sum_{k,l,m,n=0}^{\infty}\mathrm{M}^\dagger_{k,l,m,n}M_{k,l,m,n}=\sum_{k,l,m,n=0}^{\infty}\frac{\Delta_+^k\Delta_-^l\Delta_z^{(m+n+1)/2}\Pi}{k!l!m!n!}\int\frac{\mathrm{d}^2z}{\pi}|z|^{2k}z^n z^{*m}a^{\dagger l+n}e^{-a^\dagger z}|z\rangle\langle z|\mathrm{e}^{-z^* a}\mathrm{a}^{l+m}.\tag{31}$$

这样,利用相干态的密度算符的正规乘积表示[17]

$$|z\rangle\langle z|=:\exp(-|z|^2+za^\dagger+z^*a-a^\dagger a):\tag{32}$$

和积分公式

$$\int\frac{\mathrm{d}^2z}{\pi}z^k z^{*n}e^{\lambda|z|^2}=k!(-\lambda)^{-(k+1)}\delta_{n,k},\tag{33}$$

可证明广义克劳斯算符满足归一化条件,即

$$\begin{aligned}\sum_{k,l,m,n=0}^{\infty}\mathrm{M}^\dagger_{k,l,m,n}M_{k,l,m,n}&=\sum_{m,n=0}^{\infty}\frac{\Delta_z^{(m+n+1)/2}\Pi}{m!n!}\int\frac{d^2z}{\pi}\mathrm{e}^{(\Delta_+-1)|z|^2}:(za^\dagger)^n(z^*a)^m e^{(\Delta_--1)a^\dagger a}:\\&=\sum_{n=0}^{\infty}\frac{\Delta_z^{(2n+1)/2}\Pi}{n!}:a^{\dagger n}a^n e^{(\Delta_--1)a^\dagger a}:(1-\Delta_+)^{-(n+1)}=1.\end{aligned}\tag{34}$$

这意味着广义克劳斯算符 $\mathrm{M}^\dagger_{k,l,m,n}$ 和 $M_{k,l,m,n}$ 为保迹操作,即

$$\mathrm{Tr}\rho_t=\mathrm{Tr}(\sum_{k,l,m,n=0}^{\infty}M_{k,l,m,n}\rho_0\mathrm{M}^\dagger_{k,l,m,n})=\mathrm{Tr}\rho_0=1.\tag{35}$$

总之,本文利用连续变量热场纠缠态表象下系统(实模)与环境(虚模)间的算符对应关系,推导出了受振幅阻尼和热噪声同时影响的克尔介质量子主方程的解析解,即系统的密度算符随时间的解析退相干演化规律,并给出了解析解的无限维广义克劳斯算符和表示.此解析解能为进一步探查初始态的维格纳函数、光子数分布等函数以及非经典性质的时间演化提供便利.此外,还发现广义克劳斯算符满足归一化条件,且是保迹操作.

参 考 文 献

[1] Imoto N,Haus H A,Tamamoto Y. Quantum nondemolition measurement of the photon number via the optical Kerr effect[J]. Phys Rev A,1985,32(4):2287-2292.

[2] Turchette Q A,Hood C J,Lange W,et al. Measurement of conditional phase shifts for quantum logic[J]. Phys Rev Lett,1995,75(25):4710-4713.

[3] Mohapatra A K,Bason M G,Butscher B,et al. A giant electro-optic effect using polarizable dark states[J]. Nature Phys,2008,4(11):890-894.

[4] Milburn G J,Holmes C A. Quantum coherence and classical chaos in a pulsed parametric oscillator with a Kerr nonlinearity[J]. Phys Rev A,1991,44(7):4704-4711.

[5] B Yurke, D Stoler. Generating quantum mechanical superpositions of macroscopically distinguishable states via amplitude dispersion[J]. Phys Rev Lett,1986,57(1):13-16.

[6] Bužek V. Time evolution of an anharmonic oscillator in an initial Holstein-Primakoff SU(1,1) coherent state[J]. Phys Rev A,1989,39(10):5432-5435.

[7] Hu L Y,Duan Z L,Xu X X,et al. Wigner function evolution in a self-Kerr medium derived by entangled state representation[J]. J Phys A: Math Theor,2011,44(19): 195304.

[8] Wu M Y,Lv H Y,Wang J S,et al. Wigner-function evolution and photon-number decay of quantum states in a laser cavity with the Kerr medium[J]. Int J Theor Phys,2020,59(2):350-360.

[9] Román-Ancheyta R,Berrondo M,Récamier J. Parametric oscillator in a Kerr medium: evolution of coherent s tates[J]. J Opt Soc Am B,2015,32(8): 1651-1655.

[10] Kunz L,Paris M G A,Banaszek K. Noisy propagation of coherent states in a lossy Kerr medium[J]. J Opt Soc Am B,2018,35(2):214-222.

[11] Stobińska M,Milburn G J,Wódkiewicz K. Wigner function evolution of quantum states in the presence of self-Kerr interaction[J]. Phys Rev A,2008,78(1):013810.

[12] Fan H Y,Hu L Y. Operator-sum representation of density operators as solutions to master equations obtained via the entangled state approach [J]. Mod Phys Lett A,2008,22(25):2435-2468.

[13] Meng X G,Goan H S,Wang J S,et al. Nonclassical thermal-state superpositions: Analytical evolution law and decoherence behavior[J]. Opt Commun,2018,411(7):15-20.

[14] 孟祥国. 涉及双变量 Hermite 多项式的新二项式定理及应用[J]. 聊城大学学报(自然科学版),2018,31(2):66-86.

[15] 孟祥国. 量子力学纯态表象与混合态表象间的积分变换[J]. 聊城大学学报(自然科学版),2020,33(4):27-31.

[16] Fan H Y. Operator ordering in quantum optics theory and the development of Dirac's symbolic method[J]. J Opt B:Quantum Semiclass Opt,2003,5(4):R147-R164.

[17] Fan H Y,Yun H L,Fan Y. Newton-Leibniz integration for ket-bra operators in quantum mechanics and derivation of entangled state representations[J]. Ann Phys,2006,321(2):480-494.

Analytical Solution of Master Equation of the Kerr Medium Affected by Simultaneous Amplitude Damping and Thermal Noise

MENG Xiang-guo　LIU Jun-yi

(Shandong Provincial Key Laboratory of Optical Communication Science and Technology,School of Physical Science and Information Engineering,Liaocheng University,Liaocheng 252059,China)

Abstract　Using the continuous-variable thermal entangled state representations to derive the operator correspondence relations between the real mode of the system and the fictitious mode of the environment. Based on these relations,the quantum master equation describing the Kerr medium immersed in simultaneous amplitude damping and thermal noise is solved analytically, and the infinite-dimensional generalized Kraus operator-sum representation corresponding to the time-dependent density operator is presented. In addition,it is proved that the generalized Kraus operator satisfies the normalization condition and is a trace preserving operation.

Key words　thermal entangled state representation;noisy Kerr medium;generalized Kraus operator