文章编号　1672-6634(2020)05-0055-05　　**DOI**　10.19728/j.issn1672-6634.2020.05.008

量子力学纯态表象与混合态表象间的积分变换

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摘　要　在量子力学中，表象变换通常指的是两个纯态表象之间的变换，如由坐标表象变换到动量表象. 本文在纯态表象(坐标表象和动量表象)和混合态表象(Weyl-Wigner 表象)之间建立一种新的积分变换. 基于此，提出了一种获得系统密度算符 Wigner 函数的新方法，并直接导出了菲涅尔算符的 Weyl 经典对应和分数阶压缩算符的正规乘积表示.

关键词　积分变换；纯态表象；混合态表象；菲涅尔算符；分数阶压缩算符

中图分类号　O413.1　　**文献标识码**　A

由狄拉克提出的表象变换理论在量子力学中是一个基本的课题[1]，通常来说，它指的是两个不同的量子力学纯态表象之间的变换，例如，由坐标表象变换到动量表象

$$|p\rangle=\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle\langle q|p\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle\mathrm{e}^{\mathrm{i}pq},\tag{1}$$

或者反之也成立，它实际上是一种积分核为 $\mathrm{e}^{\mathrm{i}pq}/(2\pi)$ 的傅里叶积分. 基于坐标-动量相位空间中的 Wigner 算符 $\Delta(q,p)$，并考虑到它在整个空间中满足的完备性关系 $\iint_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q\Delta(q,p)=1$ 以及它的物理意义(实际上是 Wigner 算符 $\Delta(q,p)$ 的两个边缘分布)[2]

$$\int_{-\infty}^{\infty}\mathrm{d}p\Delta(q,p)=|q\rangle\langle q|=\delta(q-Q),\int_{-\infty}^{\infty}\mathrm{d}q\Delta(q,p)=|p\rangle\langle p|=\delta(p-P),\tag{2}$$

本文在纯态表象(坐标表象 $|q\rangle$ 和动量表象 $|p\rangle$)和混合态表象(Weyl-Wigner 表象)之间建立一种新型积分变换，并讨论它的具体应用. 利用有序算符内的积分法，坐标表象和动量表象的完备性关系可表示为[3]

$$\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle\langle q|=\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{\sqrt{\pi}}:\mathrm{e}^{-(q-Q)^2}:=1,\int_{-\infty}^{\infty}\mathrm{d}p\,|p\rangle\langle p|=\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{\sqrt{\pi}}:\mathrm{e}^{-(p-P)^2}:=1,\tag{3}$$

这样，Wigner 算符 $\Delta(q,p)$ 的正规排序为

$$\Delta(q,p)=\frac{1}{\pi}:\mathrm{e}^{-(q-Q)^2-(p-P)^2}:.\tag{4}$$

利用关系 $a=(Q+\mathrm{i}P)/\sqrt{2}$ 和 $\alpha=(q+\mathrm{i}p)/\sqrt{2}$，并注意到玻色算符 a 和 $a^\dagger$ 在正规排序符号 $::$ 内是对易的[4-7]，故算符 $\Delta(q,p)$ 能改写成如下形式

$$\Delta(q,p)=\frac{1}{\pi}:\mathrm{e}^{-2(a^\dagger-\alpha^*)(a-\alpha)}:\equiv\Delta(\alpha,\alpha^*).\tag{5}$$

利用有序算符内的积分法，可证明算符 $\Delta(q,p)$ 满足如下完备性关系

$$\iint_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q\Delta(q,p)=\frac{1}{\pi}\iint_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q:\mathrm{e}^{-(q-Q)^2-(p-P)^2}:=1.\tag{6}$$

从这个意义上，说明 $\Delta(q,p)$ 能构成一个混合态表象. 因此，根据 $\Delta(q,p)$ 的完备性关系，任何算符 ρ 都能被展开(即 Weyl 展开)

收稿日期：2020-03-20

基金项目：国家自然科学基金资助项目(11347026)；山东省自然科学基金(ZR2016AM03，ZR2017MA011)资助

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$$\rho = \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}q \Delta(q,p) W(q,p) , \tag{7}$$

或者利用式(5)和(7),算符 ρ 也可表示

$$\rho = 2\int \mathrm{d}^2\alpha \Delta(\alpha,\alpha^*) W(\alpha,\alpha^*) = \frac{2}{\pi}\int \mathrm{d}^2\alpha :\mathrm{e}^{-2(a^\dagger-\alpha^*)(a-\alpha)}: W(\alpha,\alpha^*) , \tag{8}$$

式中 $W(\alpha,\alpha^*) = W(q,p)$. 注意到 $\mathrm{Tr}[\Delta(q,p)\Delta(q',p')] = \frac{1}{2\pi}\delta(q-q')\delta(p-p')$,这样,可得到 $W(q,p) = 2\pi\mathrm{Tr}[\Delta(q,p)\rho]$,式中 $\mathrm{Tr}[\Delta(q,p)\rho]$ 为算符 ρ 的 Wigner 函数.

1 算符 $|q\rangle\langle q|p\rangle\langle p|$ 和 $\Delta(q,p)$ 间的积分变换

当把经典函数 $\mathrm{e}^{\lambda q+\sigma p}$ 量化为一个算符时,可采取如下三种方法

$$\begin{aligned} &\mathrm{e}^{\lambda q+\sigma p} = \mathrm{e}^{\lambda q}\mathrm{e}^{\sigma p} \to \mathrm{e}^{\lambda Q}\mathrm{e}^{\sigma P}, (\mathfrak{Q}\text{- 排序}), \\ &\mathrm{e}^{\lambda q+\sigma p} = \mathrm{e}^{\sigma p}\mathrm{e}^{\lambda q} \to \mathrm{e}^{\sigma P}\mathrm{e}^{\lambda Q}, (\mathfrak{P}\text{- 排序}), \\ &\mathrm{e}^{\lambda q+\sigma p} \to \mathrm{e}^{\lambda Q+\sigma P}, (\text{Weyl- 排序}), \end{aligned} \tag{9}$$

式中 $[Q,P] = \mathrm{i}\ (\hbar = 1)$. 这样,相应的三种量子化方案分别表示为

$$\begin{aligned} &\iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}q \mathrm{e}^{\lambda q+\sigma p}\delta(q-Q)\delta(p-P) = \mathrm{e}^{\lambda Q}\mathrm{e}^{\sigma P} = \mathfrak{Q}\mathrm{e}^{\lambda Q+\sigma P} , \\ &\iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}q \mathrm{e}^{\lambda q+\sigma p}\delta(p-P)\delta(q-Q) = \mathrm{e}^{\sigma P}\mathrm{e}^{\lambda Q} = \mathfrak{P}\mathrm{e}^{\lambda Q+\sigma P} , \end{aligned} \tag{10}$$

其中符号 $\mathfrak{Q}$ 指的是所有的坐标算符 Q 都站在所有动量算符 P 的左侧,而符号 $\mathfrak{P}$ 指的是所有的动量算符 P 都站在所有的坐标算符 Q 左侧,而 Weyl 排序依赖于 Wigner 算符,即

$$\iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}q \mathrm{e}^{\lambda q+\sigma p}\Delta(q,p) = \mathrm{e}^{\lambda Q+\sigma P} . \tag{11}$$

若用符号 $\vdots\ \vdots$ 来标记 Weyl 排序,则算符 $\mathrm{e}^{\lambda Q+\sigma P}$ 的 Weyl 排序可表示为

$$\mathrm{e}^{\lambda Q+\sigma P} = \vdots\,\mathrm{e}^{\lambda Q+\sigma P}\,\vdots , \tag{12}$$

把式(12)代入式(11)并利用有序算符内的积分法,可得到 Wigner 算符 $\Delta(q,p)$ 的 Weyl 排序,即[8]

$$\Delta(q,p) = \vdots\,\delta(p-P)\delta(q-Q)\,\vdots = \vdots\,\delta(q-Q)\delta(p-P)\,\vdots , \tag{13}$$

值得指出的是,算符 Q 和 P 在以上三种排序中都是对易的. 进一步,利用式(13)及其傅里叶变换,可导出 Wigner 算符的原始定义式,即

$$\vdots\,\delta(p-P)\delta(q-Q)\,\vdots = \iint_{-\infty}^{\infty} \frac{\mathrm{d}u\mathrm{d}v}{4\pi^2}\,\vdots\,\mathrm{e}^{\mathrm{i}(q-Q)u+\mathrm{i}(p-P)v}\,\vdots = \iint_{-\infty}^{\infty} \frac{\mathrm{d}u\mathrm{d}v}{4\pi^2}\mathrm{e}^{\mathrm{i}(q-Q)u+\mathrm{i}(p-P)v} = \Delta(q,p). \tag{14}$$

利用 Weyl 排序内的积分法可以建立以上三种排序之间的联系,即

$$\begin{aligned} |q\rangle\langle q|p\rangle\langle p| = \delta(q-Q)\delta(p-P) &= \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \mathrm{d}\lambda\mathrm{d}\sigma \mathrm{e}^{\mathrm{i}\lambda(q-Q)}\mathrm{e}^{\mathrm{i}\sigma(p-P)} \\ &= \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \mathrm{d}\lambda\mathrm{d}\sigma\,\vdots\,\mathrm{e}^{\mathrm{i}\lambda(q-Q)+\mathrm{i}\sigma(p-P)-\mathrm{i}\lambda\sigma/2}\,\vdots \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\sigma\,\vdots\,\delta(q-Q-\sigma/2)\mathrm{e}^{\mathrm{i}\sigma(p-P)}\,\vdots \\ &= \frac{1}{\pi}\,\vdots\,\mathrm{e}^{\mathrm{i}2(q-Q)(p-P)}\,\vdots , \end{aligned} \tag{15}$$

再利用式(13),我们有

$$\begin{aligned} |q\rangle\langle q|p\rangle\langle p| &= \frac{1}{\pi}\iint_{-\infty}^{\infty} \mathrm{d}p'\mathrm{d}q'\,\vdots\,\delta(p-P)\delta(q-Q)\,\vdots\,\mathrm{e}^{\mathrm{i}2(p-p')(q-q')} \\ &= \frac{1}{\pi}\iint_{-\infty}^{\infty} \mathrm{d}p'\mathrm{d}q'\Delta(q',p')\mathrm{e}^{\mathrm{i}2(p-p')(q-q')}. \end{aligned} \tag{16}$$

类似地,可有

$$|p\rangle\langle p|q\rangle\langle q| = \delta(p-P)\delta(q-Q) = \frac{1}{\pi}\,\vdots\,\mathrm{e}^{-\mathrm{i}2(q-Q)(p-P)}\,\vdots = \frac{1}{\pi}\iint_{-\infty}^{\infty} \mathrm{d}p'\mathrm{d}q'\Delta(q',p')\mathrm{e}^{-\mathrm{i}2(p-p')(q-q')}. \tag{17}$$

由式(16)和(17)可见,坐标和动量表象和 Wigner 表象之间满足新的积分变换,其积分核为 $e^{\pm i2(p-p')(q-q')}$. 因此,式(16)和(17)给出的积分变换的逆变换分别为

$$\frac{1}{\pi}\int\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{d}p \mid q\rangle\langle q \mid p\rangle\langle p \mid e^{-i2(p-p')(q-q')} = \Delta(q',p'),$$
$$\frac{1}{\pi}\int\int_{-\infty}^{\infty}\mathrm{d}q\mathrm{d}p \mid p\rangle\langle p \mid q\rangle\langle q \mid e^{i2(p-p')(q-q')} = \Delta(q',p'). \tag{18}$$

2　算符 ρ 的 Wigner 函数与 $\mathrm{Tr}(\rho \mid q\rangle\langle p \mid)/\mathrm{Tr}(\mid q\rangle\langle p \mid)$ 的新关系

利用积分变换(16)、(17)及其逆变换(18),可找到任意算符 ρ 的 Wigner 函数与 $\mathrm{Tr}(\rho \mid q\rangle\langle p \mid)/\mathrm{Tr}(\mid q\rangle\langle p \mid)$ 之间满足的新关系. 实际上,利用式(23)以及内积 $\langle q \mid p\rangle = e^{ipq}/\sqrt{2\pi}$,可得到

$$\begin{aligned}\frac{\mathrm{Tr}(\rho \mid q\rangle\langle p \mid)}{\mathrm{Tr}(\mid q\rangle\langle p \mid)} &= \sqrt{2\pi}\langle p \mid \rho \mid q\rangle e^{ipq} = 2\pi\mathrm{Tr}(\mid q\rangle\langle q \mid p\rangle\langle p \mid \rho)\\ &= 2\mathrm{Tr}[\vdots e^{i2(q-Q)(p-P)}\vdots \rho]\\ &= 2\mathrm{Tr}\left[\int\int_{-\infty}^{\infty}\mathrm{d}p'\mathrm{d}q' e^{i2(q-q')(p-p')}\vdots\delta(q'-Q)\delta(p'-P)\vdots\rho\right].\end{aligned} \tag{19}$$

由于 $\mathrm{Tr}[\vdots\delta(q'-Q)\delta(p'-P)\vdots\rho] = \mathrm{Tr}[\Delta(q',p')\rho] = \frac{1}{2\pi}W(q',p')$ 恰好为密度算符 ρ 的 Wigner 函数,则式(19)变成

$$\frac{\mathrm{Tr}(\rho \mid q\rangle\langle p \mid)}{\mathrm{Tr}(\mid q\rangle\langle p \mid)} = \frac{1}{\pi}\int\int_{-\infty}^{\infty}\mathrm{d}p'\mathrm{d}q' e^{i2(q-q')(p-p')}W(q',p'). \tag{20}$$

相应地,其逆变换为

$$W(q',p') = \frac{1}{\pi}\int\int_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q e^{-i2(q-q')(p-p')}\frac{\mathrm{Tr}(\rho \mid q\rangle\langle p \mid)}{\mathrm{Tr}(\mid q\rangle\langle p \mid)}, \tag{21}$$

这个积分表达式为计算算符 ρ 的 Wigner 函数提供了一种新的方法. 例如,对一个压缩参量为 λ 的单模压缩算符 $\rho_\lambda = e^{(a^{\dagger 2}-a^2)\lambda/2}$ [9,10],它的坐标本征态表示为[11]

$$\rho_\lambda = \int_{-\infty}^{\infty}\frac{\mathrm{d}q'}{\sqrt{\mu}} \mid \frac{q'}{\mu}\rangle\langle q' \mid,\ \mu = e^\lambda, \tag{22}$$

由此式直接推导出

$$\frac{\mathrm{Tr}(\rho_1 \mid q\rangle\langle p \mid)}{\mathrm{Tr}(\mid q\rangle\langle p \mid)} = \sqrt{2\pi}e^{ipq}\int_{-\infty}^{\infty}\frac{\mathrm{d}q'}{\sqrt{\mu}}\langle p \mid q'/\mu\rangle\langle q' \mid q\rangle = \int_{-\infty}^{\infty}\frac{\mathrm{d}q'}{\sqrt{\mu}}e^{ipq-ipq'/\mu}\delta(q'-q) = \frac{1}{\sqrt{\mu}}e^{ipq(1-1/\mu)}. \tag{23}$$

把式(23)代入式(21),可推导出压缩算符 ρ_λ 的 Wigner 函数,即

$$W(q',p') = \frac{1}{\pi\sqrt{\mu}}\int\int_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q e^{-i2(q-q')(p-p')}e^{ipq(1-1/\mu)} = \frac{2\sqrt{\mu}}{1+\mu}e^{-i2p'q'\frac{1-\mu}{1+\mu}} = \frac{e^{i2p'q'\tanh\lambda}}{\cosh\lambda/2}. \tag{24}$$

另一方面,当把式(24)代入式(7)时,可得到算符 ρ_λ 的 Weyl 排序形式,即

$$e^{(a^{\dagger 2}-a^2)\lambda/2} = \mathrm{sech}\frac{\lambda}{2}\int\int_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q e^{i2pq\tanh\lambda}\vdots\delta(p-P)\delta(q-Q)\vdots = \mathrm{sech}\frac{\lambda}{2}\vdots e^{i2PQ\tanh\lambda}\vdots. \tag{25}$$

3　菲涅尔算符的 Weyl 经典对应

对于菲涅尔算符[12,13]

$$F = \exp(\frac{iB}{2A}P^2)\exp[\frac{i}{2}(QP+PQ)\ln A]\exp(-\frac{iC}{2A}Q^2), \tag{26}$$

其中 $AD-BC=1$,它对应于经典光学中的菲涅尔光学变换,利用算符 $e^{i\lambda PQ}$ 的 $\mathfrak{P}$ 排序表示

$$e^{i\lambda PQ} = \mathfrak{P}[\exp\{-i(e^{-\lambda}-1)PQ\}], \tag{27}$$

可得到

$$e^{\frac{i}{2}(QP+PQ)\ln A} = \frac{1}{\sqrt{A}}e^{iPQ\ln A} = \frac{1}{\sqrt{A}}\mathfrak{P}e^{iPQ(1-\frac{1}{A})}. \tag{28}$$

结合式(26)和式(28),我们有

$$\begin{aligned}\mathrm{Tr}(F\,|\,q\rangle\langle p\,|) &= \langle p\,|\,\exp(\frac{\mathrm{i}B}{2A}P^2)\{\mathfrak{P}\exp[\mathrm{i}PQ(1-\frac{1}{A})]\}\exp(-\frac{\mathrm{i}C}{2A}Q^2)\,|\,q\rangle \\ &= \frac{1}{\sqrt{2\pi A}}\exp(\frac{\mathrm{i}Bp^2}{2A}-\frac{\mathrm{i}qp}{A}-\frac{\mathrm{i}Cq^2}{2A}).\end{aligned} \tag{29}$$

进而,把式(29)代入式(21)并经过简单的积分运算,可得到菲涅尔算符 F 的 Weyl 经典对应

$$\begin{aligned}W_F(q',p') &= \frac{1}{\pi\sqrt{A}}\int\int_{-\infty}^{\infty}\mathrm{d}p\mathrm{d}q\,\mathrm{e}^{-\mathrm{i}2(q-q')(p-p')}\,\mathrm{e}^{\frac{\mathrm{i}B}{2A}p^2}\,\mathrm{e}^{-\frac{\mathrm{i}qp}{A}}\,\mathrm{e}^{-\frac{\mathrm{i}Cq^2}{2A}}\,\mathrm{e}^{\mathrm{i}qp} \\ &= \frac{2}{\sqrt{A+D+2}}\exp[\frac{\mathrm{i}2Bq'^2-\mathrm{i}2Cp'^2+\mathrm{i}2(A-D)p'q'}{A+D+2}].\end{aligned} \tag{30}$$

4 分数阶压缩算符

特殊地,当 $B=\cosh\theta$, $C=-\cosh\theta$, $A=\sinh\theta$, $D=-\sinh\theta$ 时,则式(29)的右边变为

$$\frac{1}{\sqrt{\mathrm{i}2\pi\sinh\theta}}\mathrm{e}^{\frac{\mathrm{i}(q'^2+p'^2)}{2\tanh\theta}-\frac{\mathrm{i}q'p'}{\sinh\theta}}, \tag{31}$$

为了后面计算方便,式中增加了因子 $1/\sqrt{\mathrm{i}}$,这样式(30)恰好是分数阶压缩变换的积分核[14]. 同时,式(30)的右边,即菲涅尔算符 F 的 Weyl 经典对应变为

$$W_F(q',p')\rightarrow\sqrt{\frac{2}{\mathrm{i}}}\exp[\mathrm{i}(q^2+p^2)\cosh\theta+\mathrm{i}2qp\sinh\theta]=\sqrt{\frac{2}{\mathrm{i}}}\mathrm{e}^{2\mathrm{i}|\alpha|^2\cosh\theta+(\alpha^2-\alpha^{*2})\sinh\theta}. \tag{32}$$

另一方面,由式(8)可得到分数阶压缩算符的正规排序表示,即

$$\begin{aligned}2\int\mathrm{d}^2\alpha\sqrt{\frac{2}{\mathrm{i}}}\mathrm{e}^{2\mathrm{i}|\alpha|^2\cosh\theta+(\alpha^2-\alpha^{*2})\sinh\theta}\Delta(\alpha,\alpha^*) &= \frac{2}{\pi}\int\mathrm{d}^2\alpha\sqrt{\frac{2}{\mathrm{i}}}\mathrm{e}^{2\mathrm{i}|\alpha|^2\cosh\theta+(\alpha^2-\alpha^{*2})\sinh\theta}:\mathrm{e}^{-2(a^\dagger-\alpha^*)(a-\alpha)}: \\ &= \frac{2\sqrt{2}}{\pi\sqrt{\mathrm{i}}}:\int\mathrm{d}^2\alpha\,\mathrm{e}^{-2|\alpha|^2(1-\mathrm{i}\cosh\theta)+(\alpha^2-\alpha^{*2})\sinh\theta+2a^\dagger\alpha+2\alpha^*a-2a^\dagger a}: \\ &= \sqrt{\mathrm{sech}\theta}\,\mathrm{e}^{-\frac{\mathrm{i}\tanh\theta}{2}a^{\dagger 2}}:\mathrm{e}^{(\mathrm{i}\,\mathrm{sech}\theta-1)a^\dagger a}:\mathrm{e}^{\frac{\mathrm{i}\tanh\theta}{2}a^2},\end{aligned} \tag{33}$$

式中 $:\mathrm{e}^{(\mathrm{i}\,\mathrm{sech}\theta-1)a^\dagger a}:=\mathrm{e}^{a^\dagger a\ln(\mathrm{i}\,\mathrm{sech}\theta)}=\mathrm{e}^{a^\dagger a\ln\mathrm{sech}\theta}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^\dagger a}$,并使用了数学积分公式[15,16]

$$\int\frac{\mathrm{d}^2z}{\pi}\exp(\zeta\,|\,z\,|^2+\xi z+\eta z^*+fz^2+gz^{*2})=\frac{1}{\sqrt{\zeta^2-4fg}}\exp(\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}), \tag{34}$$

此式仅当 $\mathrm{Re}(\zeta\pm f\pm g)<0$, $\mathrm{Re}(\frac{\zeta^2-4fg}{\zeta\pm f\pm g})<0$ 时成立. 进一步,利用算符恒等式 $\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^\dagger a}a^2\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^\dagger a}=-a^2$,得到分数阶压缩算符的简洁形式,即

$$\mathrm{e}^{-\mathrm{i}\frac{\tanh\theta}{2}a^{\dagger 2}}\,\mathrm{e}^{\ln\mathrm{sech}\theta(a^\dagger a+\frac{1}{2})}\,\mathrm{e}^{-\mathrm{i}\frac{\tanh\theta}{2}a^2}\,\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^\dagger a}=\mathrm{e}^{-\mathrm{i}\frac{\theta}{2}(a^{\dagger 2}+a^2)}\,\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^\dagger a}. \tag{35}$$

综上,借助有序算符内的积分法,本文在纯态表象(坐标表象和动量表象)和混合态表象(Weyl-Wigner 表象)之间建立了一种新的积分变换,并由此提出了一种计算系统密度算符 Wigner 函数的新方法. 此外,给出了菲涅尔算符的 Weyl 经典对应和分数阶压缩算符的正规排序及其简洁表示.

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Integration Transformation between Pure State Representation and Mixed State Representation in Quantum Mechanics

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Abstract Representation transformation in quantum mechanics usually refers to the transform between two pure state representations,for example,from coordinate representation to momentum representation. In this paper we find the new integration transformation between the pure state representations(coordinate representation and momentum representation) and the mixed state representation(the Weyl-Wigner representation). Using this transformation,we find a new approach for obtaining Wigner function of operators,which helps us to find the Weyl classical correspondence of Fresnel operator and the normal ordering product of a fractional squeezing operator.

Key words integration transformation;pure state representation;mixed state representation;Fresnel operator;fractional squeezing operator